Errata and remarks on *Fundamentals of Convex Analysis*

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This is a list of errors and remarks on *Fundamentals of Convex Analysis* (1st edition, 2004) by Jean-Baptiste Hiriart-Urruty and Claude Lemaréchal.

1 Errors

Page 8, line 13-14: "we write $A \succ 0$ [resp. $A \succeq 0$]"
Page 14: The characterization of outer/inner semi-continuity is inaccurate. Let's label possible properties of F at x^* thus:

(1) Outer semi-continuous.

(2) For all $\epsilon > 0$, there is a neighborhood $N(x^*)$ such that $x \in N(x^*)$ implies $F(x) \subset F(x^*) + B(0, \epsilon)$

(3) Inner semi-continuous.

(4) For all $\epsilon > 0$, there is a neighborhood $N(x^*)$ such that $x \in N(x^*)$ implies $F(x^*) \subset F(x) + B(0, \epsilon)$

then the implications are actually:

- (1) $\implies$ (2) when F is locally bounded at x^*.
- (2) $\implies$ (1) when $F(x^*)$ is closed.
- (3) $\implies$ (4) when $F(x^*)$ is bounded.
- (4) $\implies$ (3) always.

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Page 48, line 2: "$y \in C$ whenever $2y_x - y \in C$."
Page 69, Fig. 5.3.1.: the label on the far right should be $\{x\} + N_C(x) = P_C^{-1}(x)$
Page 71, line 1: ξ^n should be ξ^m.
Page 96, line -3: last $\mathbb{R}^m$ should be $\mathbb{R}^n$
Page 97, line 7: "the epigraphical hull of the image $A'(\text{epi } g)$"
Page 117, Problem 2: Needs additional assumption that g is monotonically increasing.
Page 120, Problem 18: Let

$$A = \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix}, x = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix} y = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix} \tag{1}$$

then $f(\frac{1}{2}(x+y)) = 0.75625 > \frac{1}{2}(f(x) + f(y)) = 0.66976$
Page 133, Theorem 1.3.5 (iii): Add condition "σ is positive homogeneous"
Page 150, line -12: $P = \{(\xi, \eta) : \frac{1}{4}\xi^2 \le 1 - \eta\}$
Page 161, Problem 6: Change η^2 to ξ^2
Page 161, Problem 8: This question is wrong. Counterexample: let H be the x-axis in $\mathbb{R}^2$, and S be the area above a gaussian curve $y = e^{-x^2}$. Then $\overline{co}S \cap H = H, \overline{co}(S \cap H) = \emptyset$
Page 197, line -6: "where $U(x) := \{u \in U : g(x, u) = f(x)\}$"
Page 206, Problem 5: It's wrong. I have no idea how to fix it.
Page 206, Problem 6: "Show that $\nabla^2 f$"
Page 213: Just above **Example 1.1.5**, "$\frac{1}{2\|u\|^2}\|s\|^2$"
Page 225, line -8: "this is Proposition C.2.1.3"
Page 238, end of Theorem 4.1.1: "(Remark D.6.2.6)"
Page 242, Problem 1 (v), line 8: "equality holds if"
Page 243, Problem 9: This problem is wrong. Counterexample: $n = 1, f = i_{[0,\infty)}, K = [0, \infty)$, then

$$\inf_{x \in K} f(x) = 0, \min_{s \in K^\circ} f^*(s) = -\infty$$

2 Remarks

Page 19: This page is blank in the paper book and left out in the electronic book.
Page 48, line -3: This definition of L is trivial, since we can take two $y_1, y_2 \in \text{bd } C, y_1 \neq y_2$, and take two sequences $x_{k1} \to y_1, x_{k2} \to y_2$ from outside C, then $L \to 1$, thus L is always equal to 1 when the boundary of C is not a singleton.

Page 58, line -7: Lemma 4.3.1 is equivalent to **Theorem 4.3.3**, and does not have to await the proof of **Theorem 4.3.4**.
Page 174, Remark 2.1.2: In fact, the Lipschitz property of $f'(x, \cdot)$ is not needed:

$$\begin{aligned} \epsilon &< \limsup_{k\to\infty} \left| \frac{f(x+h_k) - f(x)}{t_k} - f'(x,d) \right| \\ &\leq \limsup_{k\to\infty} \left| \frac{f(x+h_k) - f(x+dt_k)}{t_k} \right| + \left| \frac{f(x+dt_k) - f(x)}{t_k} - f'(x,d) \right| \end{aligned} \tag{2}$$

For the first term, use Lipschitz to get it $\leq \frac{L}{t_k} \left| \frac{h_k}{t_k} - d \right| \to 0$. For the second term, it converges to 0 by definition of $f'(x, d)$. $\square$
Page 206, Problem D.6: All the proofs of the theorem in the literature require complex analysis.
Page 206, Problems D.10, D.11: Very easy, should have 0 stars.